

LINEAR BLENDING METHOD FOR THREE-PHASE OIL RELATIVE PERMEABILITY

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INTRODUCTION

Experimental relative permeability data for use in petroleum reservoir simulations is commonly obtained from two separate experiments, a WATER-OIL system in which the relative permeabilities of water k_{rw} and oil k_{row} are measured as functions of water saturation S_w , and a LIQUID-GAS system [with connate water] in which the relative permeabilities of oil k_{rog} and gas k_{rg} are measured as functions of total liquid saturation S_l . [The liquid saturation is the sum of the water and oil saturations and since water is connate we have $S_l = S_{wc} + S_o$].

We will have occasion to refer to the following residual saturations: (i) In the WATER-OIL system the *connate water saturation* S_{wc} , the largest water saturation for which water is immobile, i.e. $k_{rw}(S_{wc})=0$, and the *residual oil saturation in the presence of water* S_{orw} , the largest oil saturation for which oil is immobile, i.e. $k_{row}(1-S_{orw})=0$. (ii) In the LIQUID-GAS system the *critical gas saturation* S_{gc} , the largest gas saturation for which gas is immobile, i.e. $k_{rg}(1-S_{gc})=0$, and the *critical liquid saturation* S_{lc} , the largest liquid saturation for which oil is immobile, i.e. $k_{rog}(S_{lc})=0$. The critical liquid saturation S_{lc} is the sum of the connate water saturation S_{wc} from the water-oil system and the *residual oil saturation in the presence of gas* S_{org} from the liquid-gas system so that $S_{lc} = S_{wc} + S_{org}$.

Lacking three-phase experimental data [involving water, oil and gas], the use of the univariate data from these experiments in a reservoir model where bivariate data is required [three variables S_w, S_o, S_g subject to $S_w + S_o + S_g = 1$] requires some form of interpolation which is commonly accomplished as follows:

(i) The water phase relative permeability k_{rw} is normally obtained by interpolation for the water saturation S_w in the water-oil relative permeability table, essentially assuming that any gas if present behaves like oil as to its effect on water relative permeability. (ii) Likewise, the relative permeability for the gas phase k_{rg} is computed by interpolation for the total liquid saturation $S_l = S_w + S_o$ in the liquid-gas relative permeability table - essentially assuming that any additional water exceeding connate water behaves like oil as to its effect on gas relative permeability. (iii) The three-phase oil phase relative permeability k_{ro} is obtained by blending the univariate k_{row} data from the WATER-OIL system with the univariate k_{rog} data from the LIQUID-GAS system into a formulation involving two saturation variables.

STONE'S FORMULAE

The problem of deriving bivariate [three-phase] oil relative permeability from the univariate [two-phase] experiments discussed has been addressed by a number of authors including Dietrich and Bondor[1] and Stone[2,3]. Stone's second formula was normalized by Aziz and Settari[4] and the result has become the de-facto standard:

$$k_{ro} = k_{rowr} \left[\frac{k_{row}}{k_{rowr}} + k_{rw} \right] \left[\frac{k_{rog}}{k_{rowr}} + k_{rg} \right] - (k_{rw} + k_{rg}) \quad (1)$$

where $k_{rowr} = k_{row}(S_{wc})$ is the *maximum oil permeability at connate water* [from the water-oil system] used for normalization. The stipulation was added that negative values produced by the formula would indicate a "truly" unflowable situation.

The Stone formula giving $k_{ro}(S_w, S_l)$ is incompatible (i) with the very data on which it is based: it does NOT

reduce to the $k_{row}(S_w)$ of the WATER-OIL system if $S_g=0$ nor to the $k_{rog}(S_l)$ of the LIQUID-GAS system if $S_w=S_{wc}$. Furthermore (ii), its use of the quadratic term - whose intended rôle apparently is the reduction of the permeability in the three-phase region - lacks a rigorous physical basis and has been observed to cause spurious extrema for monotonic input data. Finally (iii), the probabilistic argument used by Stone muddles the distinction between statistical independence and disjointness of events.

LINEAR BLENDING METHOD

The *linear blending method* presented here estimates three-phase oil relative permeability k_{ro} as a *linear* combination of the oil relative permeability from the WATER-OIL system k_{row} and the oil relative permeability of the LIQUID-GAS system k_{rog} . The relative permeabilities of the experimental systems are evaluated at modified saturations as follows: Through the point in the three-phase plot representing the saturations is drawn a line parallel to the line connecting the connate points of both systems [$S_w=1-S_{orw}$ in the WATER-OIL system and $S_l=S_{lc}$ in the LIQUID-GAS system], and the intersection of this line with the boundary lines $S_g=0$ and $S_w=S_{wc}$ respectively yields the modified saturations S_w^{\sim} and S_l^{\sim} . The weighting coefficients multiplying the relative permeabilities of the experimental systems are the fractions of the distances along this line from the point under consideration to the point of intersection with the alternate system versus the total length of this line. This construction renders $k_{ro}=0$ along the line connecting the connate points ($S_w=1-S_{orw}$, $S_g=0$) of the WATER-OIL system and ($S_w=S_{wc}$, $S_l=S_{lc}$) of the LIQUID-GAS system. This line and lines parallel to it are expressed using a parameter ξ as follows

$$\xi = \frac{(1 - S_{orw} - S_w)}{(1 - S_{orw} - S_{wc})} - \frac{S_g}{(1 - S_{lc})} \quad (2)$$

which is constructed such that the value $\xi=0$ corresponds to the line connecting the connate points, and $\xi=1$ corresponds to the line passing through the single point ($S_w=S_{wc}$, $S_g=0$) where the k_{row} and the k_{rog} curves meet.

The blending formula *linearly* blends both systems along straight lines parallel to (2) according to the equation

$$\begin{aligned} k_{ro} &= \alpha(S_w - S_{wc})k_{row}(S_w^{\sim}) + \beta S_g k_{rog}(S_l^{\sim}) \quad (0 \leq \xi \leq 1) \\ k_{ro} &= 0 \quad (\xi < 0) \\ k_{ro} &= k_{rog}(S_l) \quad (\xi > 1) \end{aligned} \quad (3)$$

The modified saturations are the intersections of a line of constant ξ value passing through the point (S_w, S_g) under consideration with the water-oil system ($S_g=0$) and the liquid-gas system ($S_w=S_{wc}$)

$$\begin{aligned} S_w^{\sim} &= \frac{(1 - S_{lc})(S_w - S_{wc}) + (1 - S_{wc} - S_{orw})S_g}{(1 - S_{lc})} \\ S_g^{\sim} &= \frac{(1 - S_{lc})(S_w - S_{wc}) + (1 - S_{wc} - S_{orw})S_g}{(1 - S_{wc} - S_{orw})} \quad (4) \\ S_l^{\sim} &= (1 - S_g^{\sim}) \end{aligned}$$

The weighting factors are the distances along this line to the intersection with the water-oil and liquid-gas systems

$$\begin{aligned} \alpha &= \frac{(1 - S_{lc})}{(1 - S_{lc})(S_w - S_{wc}) + (1 - S_{wc} - S_{orw})S_g} \\ \beta &= \frac{(1 - S_{wc} - S_{orw})}{(1 - S_{lc})(S_w - S_{wc}) + (1 - S_{wc} - S_{orw})S_g} \end{aligned} \quad (5)$$

Negative values of ξ ($\xi < 0$) correspond to the unflowable region where $k_{ro}=0$ whereas a unit value of ξ ($\xi=1$) corresponds to the single point ($S_w=S_{wc}$, $S_g=0$) where the k_{row} and the k_{rog} curves meet. Finally, if a simulation model allows water saturations below connate water ($S_w < S_{wc}$) these are projected on the boundary of the valid region along a line parallel to (2) by resetting the water saturation to connate water and adjusting the gas saturation. The case $\xi > 1$ is a part of the infeasible region ($S_w < S_{wc}$).

The blending formula reduces to the water-oil system if $S_g=0$, and to the liquid-gas system if $S_w=S_{wc}$. The nonlinear weighting factors (5) multiplying k_{row} and k_{rog} in (3) provide a linear weighting between k_{row} and k_{rog} along lines parallel to the dividing line (2) in the three-phase plot and therefore the blending formula defines a ruled surface which is smooth everywhere throughout the three-phase plot with the possible exception of the single point ($S_w=S_{wc}$, $S_g=0$) where the k_{row} and the k_{rog} curves meet; for the blending formula to be continuous at this point, the values of the oil permeability from both experiments should be equal here.

NUMERICAL RESULTS

The three-phase plot of $k_{ro}(S_w, S_l)$ obtained using the blending method is shown in the case of the data for SPE problem II [4] and the use of linear interpolation. Figures 1 and 2 show the original data for the WATER-OIL and the LIQUID-GAS systems respectively. Figures 3 and 4 show the results obtained from the blending method and Stone's second model respectively [in the latter case negative values have been suppressed].

The left backplane represents the WATER-OIL system and the right backplane represents the LIQUID-GAS system. The blending method shows a smooth transition from flowable to unflowable whereas the Stone method shows a precipitous transition from flowable to unflowable along a convex arc. The plots were obtained with the MACSYMA[3] symbolic algebra program.

The Linear Blending Method is also available in the EXBOS reservoir simulation model. The smoother transition between the flowable and unflowable regions is beneficial.

REFERENCES

1. Stone
2. Aziz K.D. Settari
3. Macsyma

NOMENCLATURE

S_w Water saturation [Three-Phase, Water-Oil system]

S_o Oil saturation [Three-phase]

S_g Gas saturation [Three-phase]

S_l Liquid saturation [Liquid-Gas system]

S_{wc} Connate Water saturation [Water-Oil system]

S_{orw} Residual Oil saturation [Water-Oil system]

S_{lc} Critical Liquid saturation [Liquid-Gas system]

S_{gc} Critical Gas saturation [Liquid-Gas system]

k_{rw} Water relative permeability [Water-Oil system]

k_{row} Oil relative permeability [Water-Oil system]

k_{rog} Oil relative permeability [Liquid-Gas system]

k_{rg} Gas relative permeability [Liquid-Gas system]

k_{ro} Oil relative permeability [Three-Phase]

k_{rowr} Oil relative permeability at connate water [Water-Oil system]

\tilde{S}_w Modified Water saturation [blending method]

\tilde{S}_g Modified Gas saturation [blending method]

\tilde{S}_l Modified Liquid saturation [blending method]

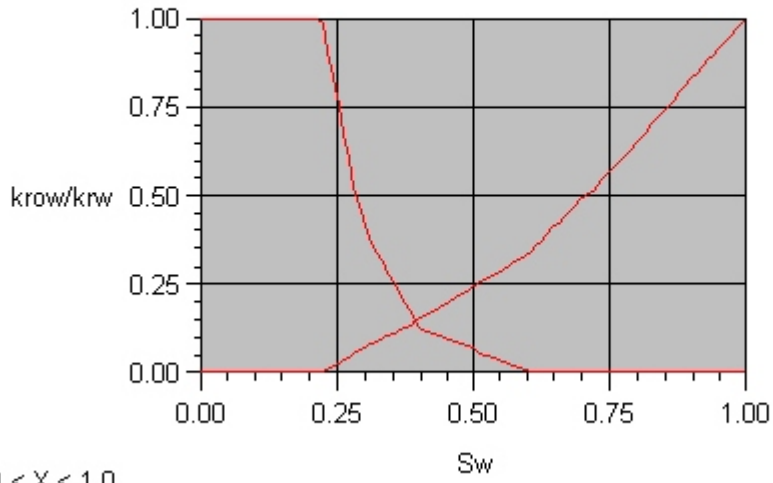
ξ Coordinate of dividing line

α Weighting factor

β Weighting factor

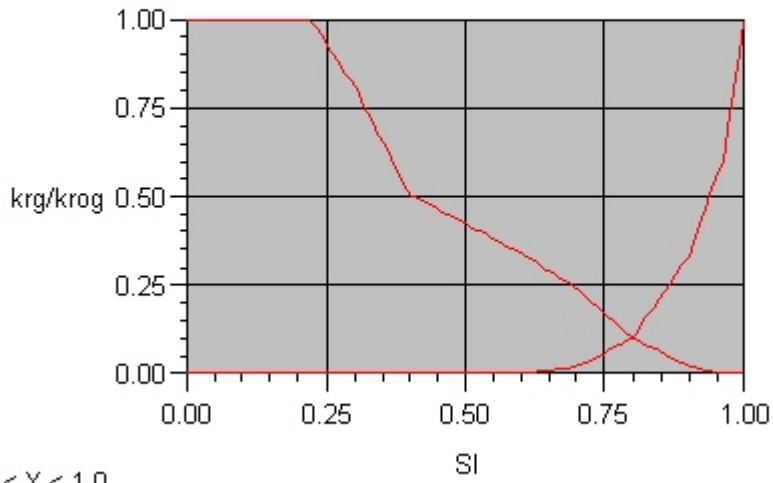
All the above quantities are dimensionless.

Relative Permeabilities Water-Oil System



$0.00 < X < 1.0$
 $0.00 < Y < 1.0$

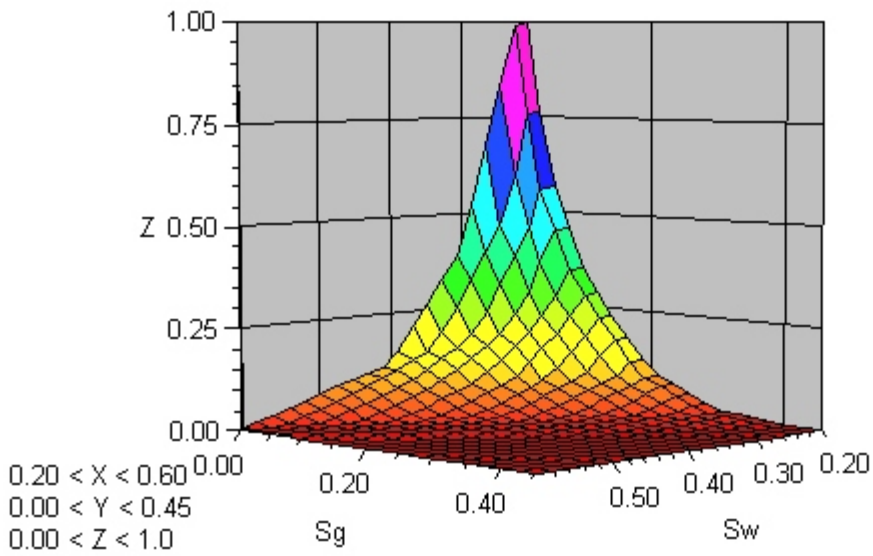
Relative Permeabilities Oil-Gas System



$0.00 < X < 1.0$
 $0.00 < Y < 1.0$

$k_{ro}(S_w, S_g)$ using Blending Method

z
 $x \perp y$



$k_{ro}(S_w, S_g)$ using Stone Model II

z
 $x \perp y$

